

The effects of metapopulation structure on multi-strain disease dynamics

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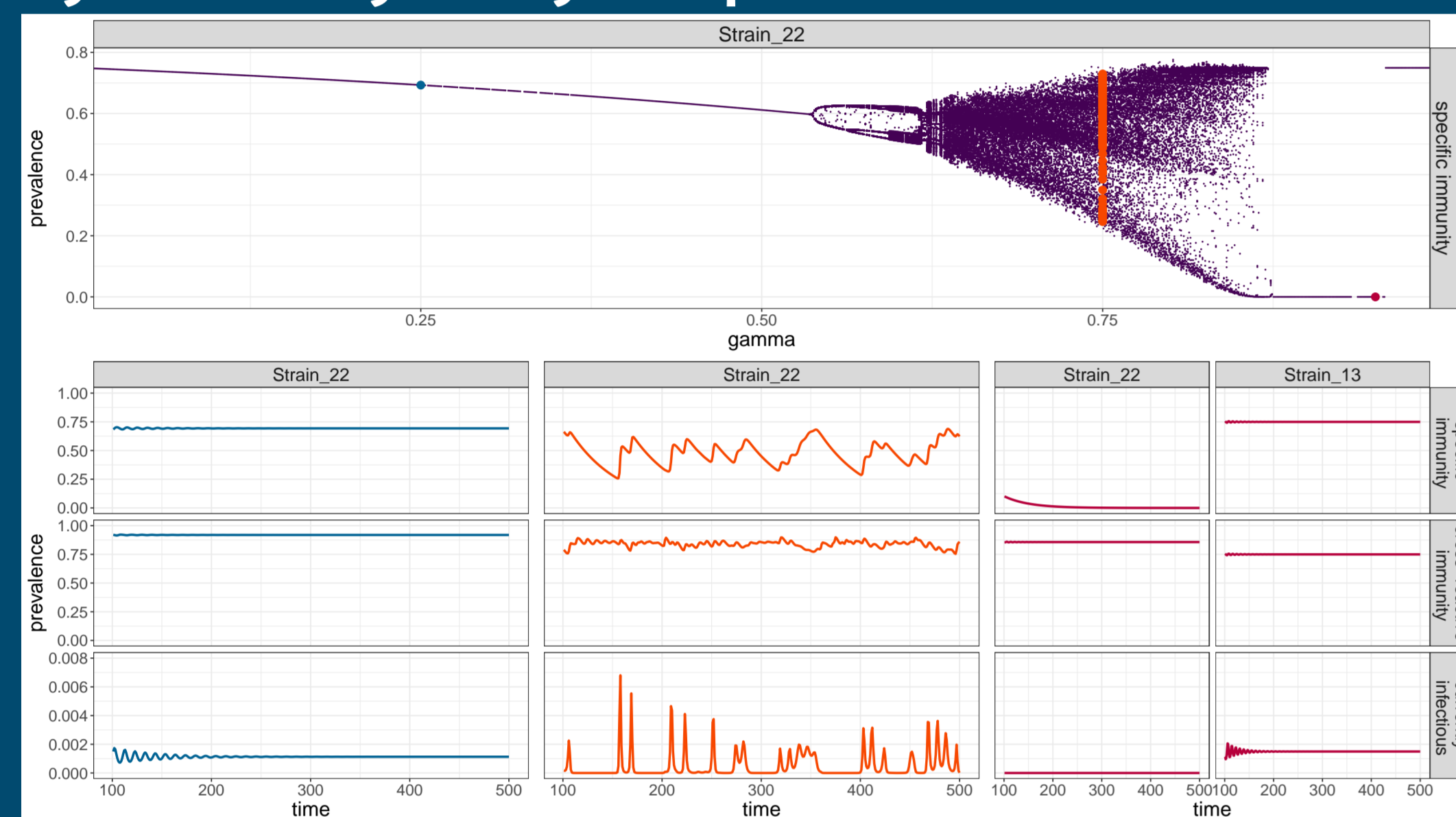
Abstract

- Many diseases have clusters in their population-genetic variability (strains)
- Exposure to one strain often yields some protection against infection by similar strains
- This partial cross-protective immunity affects disease dynamics and can influence intervention strategies.

The Gupta et al. [4] Framework

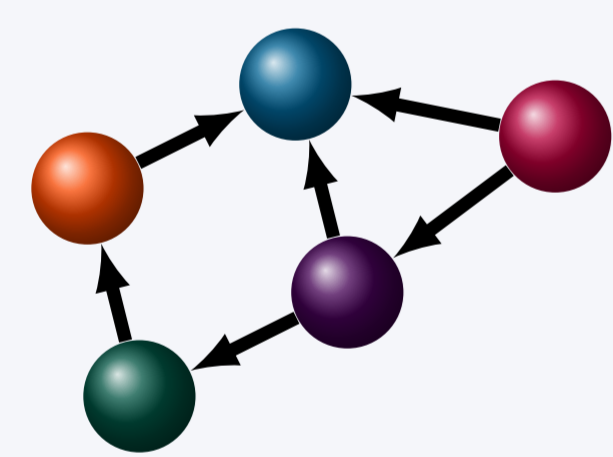
$$\begin{aligned} \frac{dy_i}{dt} &= \beta_i((1-w_i) + (1-\gamma_i)(w_i-z_i))y_i - \sigma_i y_i - \mu_i y_i \\ \frac{dz_i}{dt} &= \beta_i(1-z_i)y_i - \mu_i z_i \\ \frac{dw_i}{dt} &= \beta_i(1-w_i) \sum_{j \ni j \cap i \neq \emptyset} y_j - \mu_i w_i \end{aligned}$$

Dynamics vary widely with parameterization



Adding Migration

Following Xiao et al. [5], we model movement between populations using the matrix representation of a dispersal network:



$$\Delta = \begin{bmatrix} -3\delta & 0 & 0 & 0 & 0 \\ \delta & -\delta & 0 & 0 & 0 \\ \delta & 0 & -2\delta & 0 & \delta \\ 0 & \delta & 0 & -\delta & 0 \\ \delta & 0 & 0 & \delta & -2\delta \end{bmatrix}$$

$$\frac{dy_{i,k}}{dt} = \beta_i((1-w_{i,k}) + (1-\gamma_i)(w_{i,k}-z_{i,k}))y_{i,k} - \sigma_i y_{i,k} - \mu_i y_{i,k} + \sum_l \Delta_{kl} y_{j,l}$$

$$\frac{dz_{i,k}}{dt} = \beta_i(1-z_{i,k})y_{i,k} - \mu_i z_{i,k} + \sum_l \Delta_{kl} z_{j,l}$$

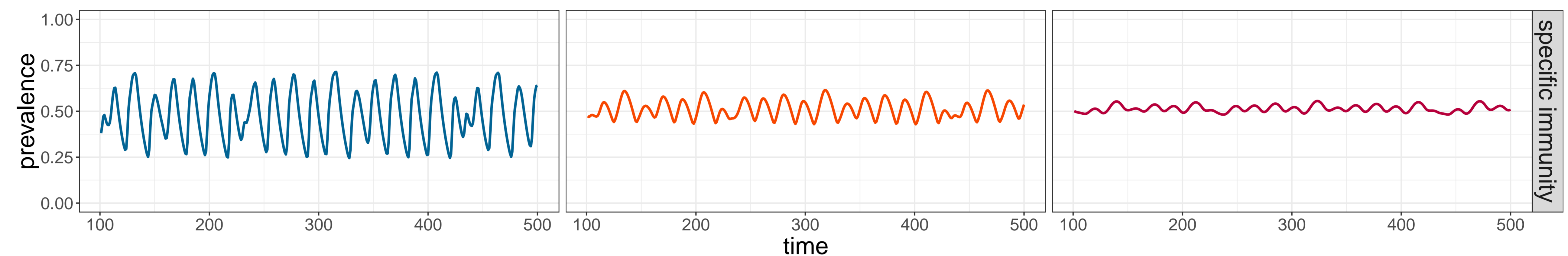
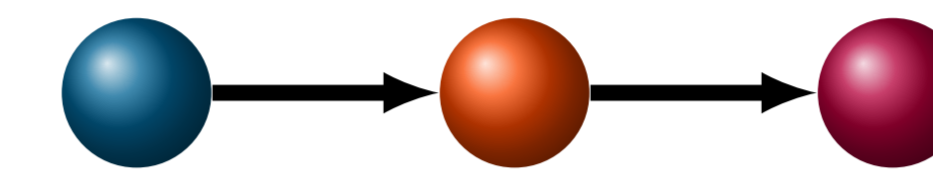
$$\frac{dw_{i,k}}{dt} = \beta_i(1-w_{i,k}) \sum_{j \ni j \cap i \neq \emptyset} y_{j,k} - \mu_i w_{i,k} + \sum_l \Delta_{kl} w_{j,l}$$

References

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- [3] This work was funded by CVM Research Office UMN Ag Experiment Station General Ag Rr search Funds.
- [4] Sunetra Gupta, Neil Ferguson, and Roy Anderson. Chaos, persistence, and evolution of strain structure in antigenically diverse infectious agents. *Science*, 280(5365):912-915, 1998.
- [5] Yanni Xiao, Yicang Zhou, and Sanyi Tang. Modelling disease spread in dispersal networks at two levels. *Mathematical medicine and biology: a journal of the IMA*, 28(3):227-244, 2011.

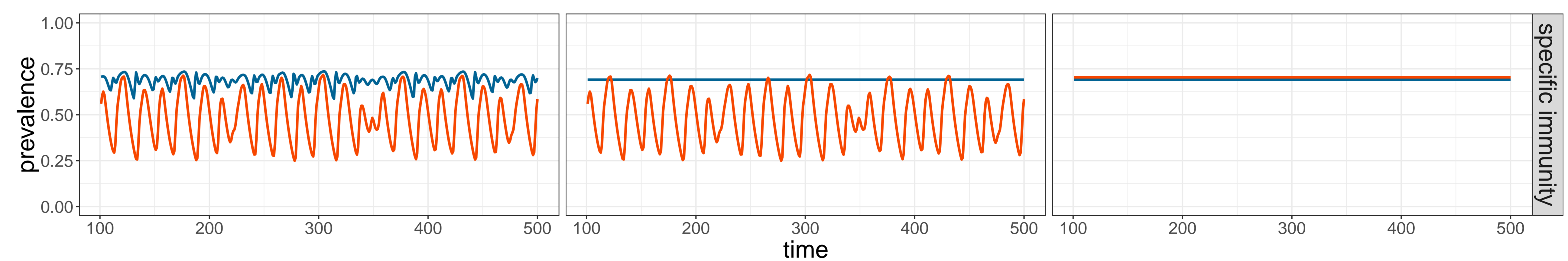
How does migration between identical populations affect dynamics?

Dynamics are dampened in populations further down the chain



How does migration affect dynamics when parameters differ between populations?

Destination populations inherit the dynamics of migrant origins



When there are multiple sources contributing to a single sink, which dynamics dominate?

Chaos overrules cycles, and cycles overrule steady states

